PROBLEM SET 9

Problem 1

Smoke-stack exhaust. A power company releases 1 kg/s of CO_2 from a height of 30 m into a wind with average velocity 4 m/s. The sky is partly cloudy and the terrain down-wind of the release is pasture land. Estimate the turbulence intensities and find the maximum concentration at ground level downstream of the release. How do the results change if the release point is lowered by 15 m?

SOLUTION:

We assume daytime conditions, with moderate solar radiation. Using Table 6.1, we find that when u = 4 m/s, this corresponds to moderately to slightly unstable conditions.

Then, based on Table 6.2, we estimate $i_y = 0.3$ and $i_z = 0.1$.

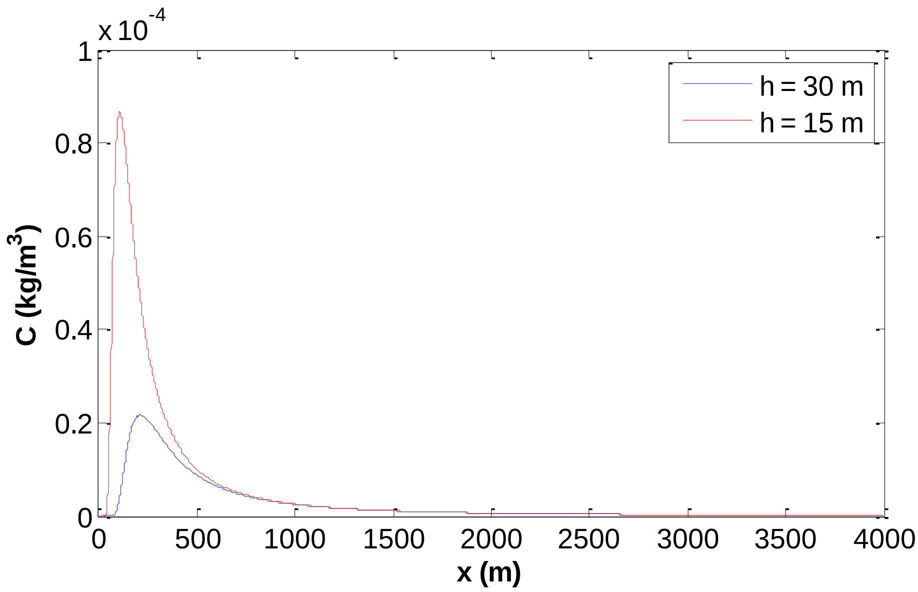
Using the near-field solution, we can determine:

$$\sigma_y = i_y x = 0.3x$$
$$\sigma_z = i_z x = 0.1x$$

In the solution for the centerline of the plume at ground level:

$$C(x,0,0) = \frac{\dot{m}}{\pi \bar{u} \sigma_y \sigma_z} \exp\left(-\frac{h^2}{2\sigma_z^2}\right)$$

Plotting this:



For, h = 30 m, $C_{max} = 2.17\text{x}10^{-5}$ kg/m³ and the maximum occurs at x = 212.1 m.

For h = 15 m, the maximum concentration is larger and it reaches the ground closer to the smoke-stack exhaust. Values are $C_{max} = 8.67 \text{ x } 10^{-5}$ kg/m³ at x = 106.1 m.

Problem 2

Accidental kerosene spill. A tanker truck has an accident and spills 100 kg of kerosene into a river. The spill occurs over a span of 3 minutes and can be approximated as uniformly distributed across the lateral cross-section of the river. A fish farm has its water intake 2.5 km downstream of the spill location. Refer to Figure 3.9.

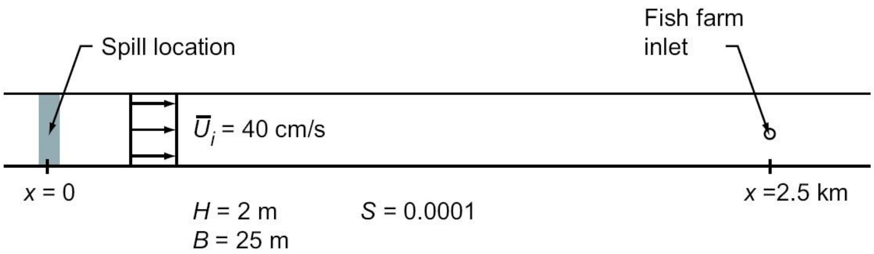


Fig. 3.9. Schematic of the accidental spill with the important measurement values. B and H are the width and depth of the river, $\overline{U_i}$ is the average river flow velocity at the accident location, and S is the channel slope.

a) Compute the dispersion coefficient from the equation from Fischer et al. (1977)

$$D_L = 0.011\frac{U^2B^2}{Hu_*}$$

where U is the mean velocity and u_* is the shear velocity. ($u_* = \sqrt{gHS}$ Equation 3.6 in the textbook)

b) Use the following relationship to compute the longitudinal dispersion coefficient, D_L:

$$\frac{D_L}{Hu_*} = \frac{0.15}{8\varepsilon_{t0}}\left(\frac{B}{H}\right)^{5/3}\left(\frac{U}{u_*}\right)^2$$

B and H are the river width and depth, U is the average flow velocity, u_* is the shear velocity, and ε_{t0} is a non-dimensional number given by:

$$\varepsilon_{t0} = 0.145 + \left(\frac{1}{3520}\right)\left(\frac{U}{u_*}\right)\left(\frac{B}{H}\right)^{1.38} = 0.229$$

c) Plot the concentration in the river as a function of downstream distance at t = 2 hr after the accident. From the figure, determine the location of the center of mass of the kerosene cloud, the maximum concentration in the river, and the characteristic width of the cloud in the x-direction (approximate the cloud using one standard deviation of the concentration distribution).

d) Write the equation for the concentration as a function of time at the inlet to the fish farm. Plot your equation and determine at what time the maximum concentration passes the fish farm.

SOLUTION:

a) $u_* = \sqrt{gHS} = 0.0443\ m/s$

From Eq. (1), we can get

$$D_L = 0.011\frac{U^2B^2}{Hu_*} = 0.011\frac{(0.4m/s)^2(25m)^2}{(2m)(0.0443m/s)} = 12.41\ m^2/s$$

b)
$$\varepsilon_{t0} = 0.145 + \left(\frac{1}{3520}\right)\left(\frac{U}{u_*}\right)\left(\frac{B}{H}\right)^{1.38} = 0.145 + \left(\frac{1}{3520}\right)\left(\frac{0.4m/s}{u_*}\right)\left(\frac{25m}{2m}\right)^{1.38} = 0.229$$

$$u_* = 0.0442\ m/s$$

$$\frac{D_L}{Hu_*} = \frac{0.15}{8\varepsilon_{t0}}\left(\frac{B}{H}\right)^{\frac{5}{3}}\left(\frac{U}{u_*}\right)^2$$

$$D_L = Hu_*\frac{0.15}{8\varepsilon_{t0}}\left(\frac{B}{H}\right)^{\frac{5}{3}}\left(\frac{U}{u_*}\right)^2 = 39.951m^2/s$$

c) From table (2.1) in the textbook, the solution of the advection-diffusion equation for instantaneous point source is shown below

$$C(x,t) = \frac{M}{A\sqrt{4\pi D_L t}}\exp\left(-\frac{(x-U\times t)^2}{4D_L t}\right)$$

When t = 2 hr, the maximum concentration will appear where

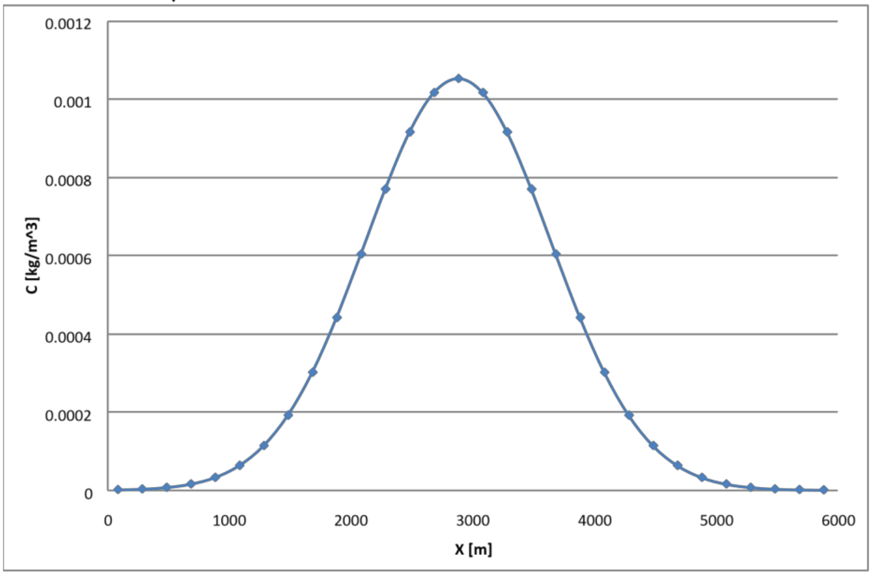
$$\frac{\partial C(x,t=2hr)}{\partial x} = 0$$

$$\frac{\partial C(x,t=2hr)}{\partial x} = \frac{M}{A\sqrt{4\pi D_L t}}\exp\left(-\frac{(x-U\times t)^2}{4D_L t}\right)\times\left(-\frac{2(x-U\times t)}{4D_L t}\right) = 0$$

$$x = U\times t = \frac{0.4m}{s} * 2hr * \frac{3600s}{hr} = 2880m$$

$$C(x=2880, t=2hr) = \frac{M}{A\sqrt{4\pi Dt}} = \frac{100}{2\times 25\sqrt{4\pi 39.95(2\times 3600)}} = 1.052\times 10^{-3}kg/m^3$$

$$\sigma = \sqrt{2D_L t} = \sqrt{2 \times 39.95 \times 2 \times 3600} = 758.48\, m$$

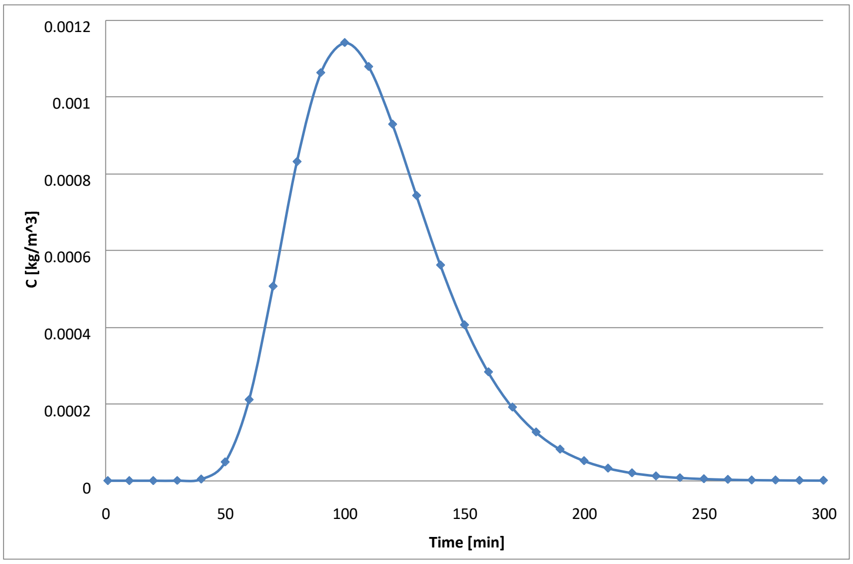


d)

$$C(x_{fish}, t) = \frac{M}{A\sqrt{4\pi D_L t}} \exp\left(-\frac{(x_{fish} - U \times t)^2}{4D_L t}\right) = \frac{M}{A\sqrt{4\pi D_L t}} \exp\left(-\frac{(2500 - U \times t)^2}{4D_L t}\right)$$

$$\frac{\partial C(x_{fish} = 2500, t)}{\partial t} = 0$$

$$\frac{\partial C(x_{fish} = 2500, t)}{\partial t} = 0 \rightarrow t = \frac{\sqrt{D_L^2 + U^2 x_{fish}^2} - D_L}{U^2} = 6005\, s \approx 100\, min$$


0.0012
0.001
0.0008
0.0006
0.0004
0.0002
0
C [kg/m^3]
0
50
100
150
200
250
300
Time [min]